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Combined GPS and BeiDou instantaneous RTK positioning

ROBERT ODOLINSKI
GNSS Research Centre, Curtin University

PETER J.G. TEUNISSEN
GNSS Research Centre, Curtin University, and
Mathematical Geodesy and Positioning, Delft University of Technology

DENNIS ODIJK
GNSS Research Centre, Curtin University

ABSTRACT

The Chinese BeiDou-2/COMPASS Navigation Satellite System has attained regional operational status and is expected to reach the same level of popularity as GPS once it has reached its full constellation. This contribution considers combined BeiDou+GPS relative code positioning and single- and multiple-frequency real-time kinematic (RTK) positioning. A combined system increases the redundancy for solving the unknown GNSS parameters and thus allows for more precise position estimates, improved reliability and robustness against failure of any of the systems. The performance is evaluated by ambiguity success-rates and by comparing the estimated positions to very precise benchmark coordinates. We make use of the LAMBDA method for integer ambiguity resolution, in combination with the Fixed Failure-rate Ratio Test to validate the resolved ambiguities. The combined model will be shown to allow for improved ambiguity resolution performance and positioning robustness and accuracy over the BeiDou- and GPS-only solutions.

Keywords: Positioning, Navigation and Timing (PNT), LAMBDA, Success rates, Fixed-Failure rate Ratio Test (FFRT)

INTRODUCTION

The full BeiDou (COMPASS) Navigation Satellite System constellation is expected in 2020, and will consist of five Geostationary Earth Orbit (GEO), three Inclined Geo-Synchronous Orbit (IGSO) and 27 Medium Earth Orbit (MEO) satellites [1]. BeiDou attained initial regional operational status in the end of December 2011, and can provide Positioning, Navigation and Timing (PNT) services in the whole Asia-Pacific region. Besides BeiDou simulation results in e.g. [2–9], we have real data analysis in [10] that processed BeiDou Single Point Positioning (SPP), relative code positioning, and BeiDou Real-Time-Kinematic (RTK). Moreover [11] evaluated orbit determination for BeiDou and combined BeiDou+GPS Precise Point Positioning (PPP). First results using BeiDou outside of China are reported in [12–15] that considered orbit determination, PPP, and single-baseline RTK positioning.

In this contribution we present combined BeiDou+GPS relative code positioning, here also referred to as Relative Point Positioning (RPP), and single-baseline RTK positioning results, and a comparison is made to the systems separately. We focus on the challenging case of single-epoch ambiguity resolution, which has the advantage of being insensitive to cycle-slips. The models considered are of the single-baseline type using two receivers separated by a short distance. Hence, the residual ionospheric and tropospheric delays as well as satellite orbit errors are assumed negligible.

We start with an overview of the BeiDou system and then describe the between-receiver single-differenced (SD) RTK functional and stochastic model used. This follows by an outline of the methods used for ambiguity resolution and validation. For ambiguity resolution the LAMBDA method [16–20] is used in combination with the Fixed Failure-rate Ratio Test [21, 22]. Results are then given for ambiguity success rates, RPP and RTK positioning. We conclude with a summary and discussion.

Based on presentation given at *Institute of Navigation PNT Conference, Honolulu, Hawaii, April 23-25, 2013*

BEIDOU SATELLITE SYSTEM

We depict in Figure 1 a 24 hour ground track of the operational BeiDou satellites available for positioning at July 7, 2012 as visible from the Curtin University station CUT0 in Western Australia, Perth. The satellites location is presented as a dot at UTC 4:25 as observed from the receiver at this time-instance. The satellite constellation consists of four GEO satellites at an altitude of 35,786 km, five IGSO satellites at an altitude of 35,786 km as well, with 55 degree inclination to the equatorial plane, and two MEO satellites at an altitude of 21,528 km with same inclination [1].

The figure shows that the IGSO and MEO satellites are located to the South of CUT0 for some periods, whereas the remaining GEO satellites always remain stationary in the North around the equator. The GEO satellites are controlled in longitude but not latitude and therefore reach peak inclinations of about 2 degrees [14], and the IGSO satellites describe figure-of-eight loops.

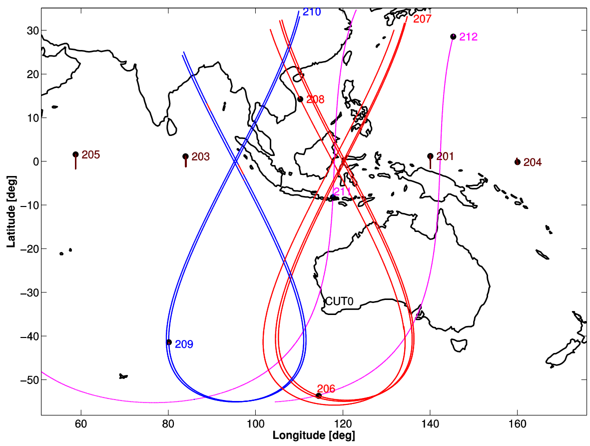

Figure 1: BeiDou ground track from Perth station CUT0 given at July 7, 2012, and satellite locations are given as a dot at UTC 4:25.

The BeiDou signals are based on Code Division Multiple Access (CDMA) similar to GPS, Galileo and QZSS. BeiDou satellites transmit at three frequencies, B1, B2 and B3 in Quadrature phase-shift keying (QPSK) modulation [23] as is shown in Table 1 and given with the L1, L2 and L5 GPS frequencies. Currently (2013) none of the BeiDou frequencies overlap the GPS ones.

Table 1: BeiDou/COMPASS and GPS signals.

Sat. system	Band	Frequency [MHz]	Wavelength [cm]
BeiDou	B1	1561.098	19.20
	B2	1207.140	24.83
	B3	1268.520	23.63
GPS	L1	1575.42	19.03
	L2	1227.60	24.42
	L5	1176.45	25.48

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SYSTEM OF GNSS OBSERVATION EQUATIONS

Consider receivers $r = 1,2$ tracking the GPS or BeiDou satellites $s_* = 1_*, \ldots, m_*$, where m_* is the number of satellites of system $*$ (G for GPS and C for COMPASS). Since there are no overlapping frequencies between GPS and BeiDou we define them as $j_* = 1_*, \ldots, f_*$, where f_* is the number of frequencies for system $*$. External products for satellite orbits are used and between-receivers single-differencing (SD) is subsequently performed on the system of observation equations with respect to the 'pivot' receiver 1. The satellite delays common to the receivers are then eliminated and for short baselines of a few km, satellite orbit errors, relative Zenith Tropospheric Delay and ionosphere can be ignored as well.

Full-rank RTK functional model

The system of equations is however not of full-rank after the SDs. These singularities or rank defects can be eliminated through an application of S-system theory [24–28], implying null-space identification, S-basis constraining and proper interpretation of the estimable parameters.

After solving the rank deficiencies we have the following *single-system*, single-differenced (SD) full-rank (linearized) observation equations in units of range, and we omit time stamps for brevity,

$$\begin{aligned} p^{s_*}_{12,j_*} &= -c_2^{s_* T}\Delta x_{12} + d\tilde{t}_{12} + \tilde{d}_{12,j_*} \\ \phi^{s_*}_{12,j_*} &= -c_2^{s_* T}\Delta x_{12} + d\tilde{t}_{12} + \tilde{\delta}_{12,j_*} + \lambda_{j_*}\tilde{M}^{1 s_*}_{12,j_*} \end{aligned} \qquad (1)$$

where $(\cdot)_{12} = (\cdot)_2 - (\cdot)_1$ is the notation for between-receiver SDs, the SD code and phase observable is denoted $p^{s_*}_{12,j_*}$ and $\phi^{s_*}_{12,j_*}$ respectively, where $c_r^{s_* T} = \frac{(x^{s_*} - x_r)^T}{||x^{s_*} - x_r||}$ is the line-of-sight unit vector from the receiver r to the satellites obtained from linearizing the system of equations with respect to the receiver coordinates, and λ_{j_*} the wavelength corresponding to frequency j_*. We refrain from carrying through random observation noise and other un-modeled effects such as multipath explicitly for notational convenience. The interpretation of the estimable parameters is as follows,

$\Delta x_{12} = \Delta x_2 - \Delta x_1$	Relative receiver coordinates,
$d\tilde{t}_{12} = dt_2 + d_{2,1_*} - (dt_1 + d_{1,1_*})$	Relative receiver clock with code delays on $j_* = 1_*$,
$\tilde{d}_{12,j_*} = d_{2,j_*} - d_{2,1_*} - (d_{1,j_*} - d_{1,1_*})$	Relative Differential Code Bias (DCB), $j_* = 2_*, \ldots, f_*$,
$\tilde{\delta}_{12,j_*} = \delta_{2,j_*} - \delta_{1,j_*} - (d_{2,1_*} - d_{1,1_*}) + \lambda_{j_*} M^{1_*}_{12,j_*}$	Relative receiver hardware phase delays,
$\tilde{M}^{1 s_*}_{12,j_*} = M^{s_*}_{2,j_*} - M^{s_*}_{1,j_*} - M^{1_*}_{2,j_*} + M^{1_*}_{1,j_*}$	Double-differenced ambiguities, $s_* \geq 2$.

The full-rank system of observation equations for a *combined system* can be expressed in matrix-vector form as

$$y = Aa + Bb \tag{2}$$

with the code- and phase observation vector y of dimension $2f_G m_G + 2f_C m_C$, given as $y = [y_p^{G\,T}, y_\phi^{G\,T}, y_p^{C\,T}, y_\phi^{C\,T}]^T$, where $(.)^T$ denotes the transpose of a vector, $y_p^* = [y_{p;1_*}^{*\,T}, \ldots, y_{p;f_*}^{*\,T}]^T$, $y_{p;j_*}^* = [p_{12,j_*}^{1_*}, \ldots, p_{12,j_*}^{m_*}]^T$, $y_\phi^* = [y_{\phi;1_*}^{*\,T}, \ldots, y_{\phi;f_*}^{*\,T}]^T$, $y_{\phi;j_*}^* = [\phi_{12,j_*}^{1_*}, \ldots, \phi_{12,j_*}^{m_*}]^T$, and with ambiguity (a) design matrix

$$A = \text{blkdiag}(\left[0^T \Lambda_G^T \otimes C_{m_G}^T\right]^T, \left[0^T \Lambda_C^T \otimes C_{m_C}^T\right]^T) \tag{3}$$

The remaining unknowns (b) corresponding design matrix reads $B = [B_1, B_2, B_3]$, where

$$\begin{aligned} B_1 &= \left[(e_2 \otimes e_{f_G} \otimes G^G)^T, (e_2 \otimes e_{f_C} \otimes G^C)^T\right]^T \\ B_2 &= \text{blkdiag}\left(e_2 \otimes e_{f_G} \otimes e_{m_G}, e_2 \otimes e_{f_C} \otimes e_{m_C}\right) \\ B_3 &= \text{blkdiag}\left(C_{f_G} \otimes e_{m_G}, I_{f_G} \otimes e_{m_G}, C_{f_C} \otimes e_{m_C}, I_{f_C} \otimes e_{m_C}\right) \end{aligned} \tag{4}$$

We have e_2, e_{f_*} and e_{m_*} as the column vectors with only ones of size 2×1, $f_* \times 1$ and $m_* \times 1$ respectively, G^* contains the receiver-satellite line-of-sight unit vectors, $\otimes$ is the Kronecker product [29], 'blkdiag' denotes a blockdiagonal matrix, I_{f_*} the identity matrix of size f_*, $C_{f_*} = [0_{1\times(f_*-1)}^T, I_{(f_*-1)}^T]^T$, $C_{m_*} = [0_{1\times(m_*-1)}^T, I_{(m_*-1)}^T]^T$, and $\Lambda_* = \text{diag}(\lambda_{1_*}, \ldots, \lambda_{f_*})$ the diagonal wavelength matrix. We have the same interpretation of the estimable unknowns in (2) as in (1), where the GPS clock is with respect to GPS time and the BeiDou clock to BeiDou navigation satellite system time [1].

RTK stochastic model

The variance-covariance (VCV) matrix of the code and phase observables in SD form and for a single-system (1) is given as,

$$Q_{yy}^* = \text{blkdiag}\left(C_p^*, C_\phi^*\right) \otimes \left(D_n^T D_n \otimes W_{m_*}^{-1}\right) \tag{5}$$

where D_n^T is the between-receivers SD operator [30], with -1 for the pivot receiver and a 1 for the second receiver, and the code and phase observable a priori variance factors are given in the sub-matrices $C_p^* = \text{diag}(\sigma_{p,1_*}^2, \ldots, \sigma_{p,f_*}^2)$ and $C_\phi^* = \text{diag}(\sigma_{\phi,1_*}^2, \ldots, \sigma_{\phi,f_*}^2)$ respectively. We assume no cross-correlation between code and phase nor between frequencies. Further $W_{m_*}^{-1}$ contains the exponential elevation-weighting function defined by [31]. The VCV-matrix of the combined system (2) is given as,

$$Q_{yy} = \text{blkdiag}\left(Q_{yy}^G, Q_{yy}^C\right) \tag{6}$$

Redundancy and solvability

The redundancy is the number of observations minus the number of estimable unknowns. In Table 2 we give the number of observations, the number of estimable unknowns and the redundancy for the single-baseline RTK models (1) and (2). In the last column a *solvability condition* is depicted as well, which is the minimum number of satellites required to solve the models.

Table 2: Single-baseline RTK, number of observations, unknowns, redundancy and solvability condition.

Model	**# of observations**	**# of unknowns**	**Redundancy**	**Solvability condition**
Single-baseline RTK				
Single-system (1)	$2f_* m_*$	$3 + f_* + f_* m_*$	$f_*(m_* - 1) - 3$	$m_* \geq 4$
Combined (2)	$2f_G m_G + 2f_C m_C$	$3 + f_G + f_G m_G + f_C + f_C m_C$	$f_G(m_G - 1) + f_C(m_C - 1) - 3$	$m_G + m_C \geq 5$

FLOAT AND FIXED LEAST-SQUARES SOLUTION

Consider the observation equations in (1) or (2) with ambiguities in vector a of size $q \times 1$ and the other unknowns in b of say size $p \times 1$. We will solve the following (mixed) Integer Least-Squares (ILS) problem [16],

$$\min_{b,a} ||y - Bb - Aa||_{Q_{yy}}^2, \text{ with } b \in \mathbb{R}^p, a \in \mathbb{Z}^q \tag{7}$$

where $||\cdot||_{Q_{yy}}^2 = (\cdot)^T Q_{yy}^{-1} (\cdot)$, $\mathbb{R}^p$ is the p-dimensional space of real numbers and $\mathbb{Z}^q$ the q-dimensional space of integers. The parameter estimation is divided into the three following steps.

Float solution

In the float solution we replace the integer constraint $\mathbb{Z}^q$ in (7) with $\mathbb{R}^q$, i.e. the ambiguities will also be estimated as real-valued parameters. The unknown parameters can be solved by least-squares as follows [32],

$$\begin{bmatrix} \hat{b} \\ \hat{a} \end{bmatrix} = \begin{bmatrix} Q_{\hat{b}\hat{b}} & Q_{\hat{b}\hat{a}} \\ Q_{\hat{a}\hat{b}} & Q_{\hat{a}\hat{a}} \end{bmatrix} \begin{bmatrix} B^T Q_{yy}^{-1} y \\ A^T Q_{yy}^{-1} y \end{bmatrix} \tag{8}$$

where $\hat{a}$, $\hat{b}$ are the least-squares solution of the ambiguities and baseline components respectively, and $Q_{\hat{a}\hat{a}}$, $Q_{\hat{b}\hat{b}}$, $Q_{\hat{a}\hat{b}}$, $Q_{\hat{b}\hat{a}}$ are the corresponding (co)variance matrices.

Integer ambiguity estimation

It can be shown that the integer solution of (7) is given by the integer minimizer,

$$\check{a} = \arg\min_{z \in \mathbb{Z}^q} ||\hat{a} - z||_{Q_{\hat{a}\hat{a}}}^2 \tag{9}$$

and can efficiently be computed with the LAMBDA method [16]. The ILS estimator has the highest possible success rate of all integer estimators [33–35].

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by the GNSS centre of Wuhan University for this period. For GPS, standard broadcast ephemeris are used, and the estimated positions are compared to very precise station benchmarks. We make use of the Detection, Identification and Adaptation (DIA) procedure to eliminate outliers [36].

A BeiDou skyplot and typical number of satellites and Positional Dilution Of Precision (PDOP) for a combined BeiDou+GPS system is given in Figure 2 for July 7, 2012, in UTC +0 hours. We see close to twice the number of satellites when combining the systems as compared to using the systems separately.

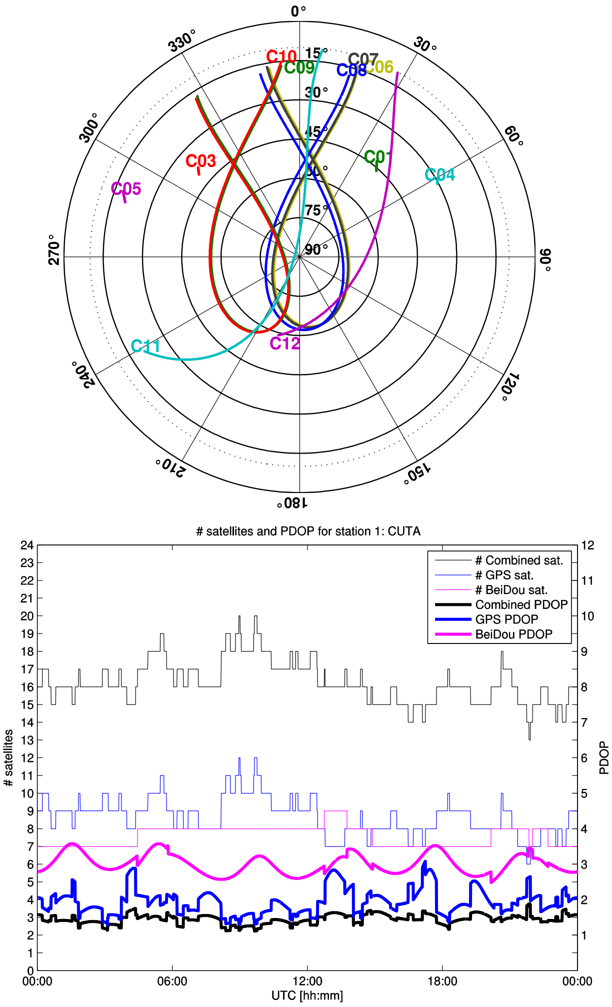

Figure 2: BeiDou skyplot (top) and satellite visibility/PDOP (bottom) for combined BeiDou+GPS and 10 degrees elevation cut-off angle (CUTA, July 7 2012).

Stochastic model settings

The stochastic model settings for RTK positioning are given in Table 3. To find the a priori code and phase

Fixed baseline solution

The fixed baseline solution $\check{b}$ of the (integer) constrained linear model is given as,

$$\check{b} = \hat{b} - Q_{\hat{b}\hat{a}} Q_{\hat{a}\hat{a}}^{-1} (\hat{a} - \check{a}) \tag{10}$$

The uncertainty in $\check{a}$ can be neglected if the probability of correct integer estimation is sufficiently high.

Integer validation: Fixed Failure rate Ratio Test

The ratio-test reads,

$$\text{accept } \check{a} \text{ if } \frac{||\hat{a} - \check{a}||^2_{Q_{\hat{a}\hat{a}}}}{||\hat{a} - \check{a}'||^2_{Q_{\hat{a}\hat{a}}}} \leq c \tag{11}$$

where $\check{a}'$ is the integer vector that gives the next smallest value of the quadratic form and c is the critical value of the test. In practice one often still works with a constant value for c. This, however, is nonoptimal as explained and demonstrated in [21, 22]. Instead of a constant value for c, a variable value should be used based on a user-defined failure-rate P_f. This is the concept of the Fixed Failure-rate Ratio Test (FFRT), where the chosen P_f is used to compute the required value for c. The classical approach that uses a constant value for c is referred to as the Fixed Critical-value Ratio Test (FCRT).

If we denote the probability of correct integer estimation as P_s, then $P_s + P_f$ is the acceptance probability of test (11), and the probability of successful fixing follows as,

$$P_{sf} = \frac{P_s}{P_s + P_f} \tag{12}$$

This shows that by computing the variable c from a user-defined constant, but small, failure-rate P_f, the probability of successful fixing P_{sf} can be made large. This implies that one can be confident that any integer solution that passes the FFRT is indeed correct. Would one, on the other hand, work with a constant value for c, then no such constant quality of the ambiguity resolution process can be guaranteed.

AMBIGUITY SUCCESS-RATE RESULTS

In this section the integer ambiguity success rates for combined BeiDou+GPS and BeiDou- and GPS-only are presented. Data from Curtin's Continuously Operating Reference Stations was used. The stations are equipped with Trimble NetR9 multi-frequency multi-GNSS receivers and has an inter-station distance of approximately 10 m. Five days of data is analyzed, namely July 7-11, 2012, with 10 degrees of elevation cutoff angle and 30 sec interval between consecutive measurements. The BeiDou satellite orbit and clock products are provided

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standard deviation settings we used one day of data to estimate the standard deviations. We then applied these settings to the other days to independently check the validity of the stochastic model, see further description in relation to Figure 4.

Table 3: Zenith referenced stochastic model settings for undifferenced code- and phase observables.

	Frequency	Code $\sigma_{p,j*}$ [cm]	Phase $\sigma_{\phi,j*}$ [mm]
GPS	L1	37	2
	L2	27	2
BeiDou	B1	31	2
	B2	30	2
	B3	25	2

Integer ambiguity success rates

We now evaluate the success rate performance of single-baseline RTK for a combined BeiDou+GPS system and the single-systems separately. The empirical success rate is computed by comparing the single-epoch estimated ambiguities to reference ambiguities, estimated by using a combined system with multiple-frequencies and a Kalman filter over the whole time span and assuming the ambiguities time-constant. The empirical ILS success rate is defined as,

$$P_{s_E} = \frac{\text{\# of correctly fixed epochs}}{\text{total \# of epochs}} \tag{13}$$

The empirical ILS failure rate is given as the complement,

$$P_{f_E} = 1 - P_{s_E} \tag{14}$$

These two measures are evaluated without any integer validation, i.e. it is the outcome if we would accept all integer ambiguities. The accepted solutions by the FFRT (11) can also be compared to the reference ambiguities accordingly. The empirical success rate for the FFRT test follows as,

$$P_{s,FFRT} = \frac{\text{\# of accepted and correctly fixed epochs}}{\text{total \# of epochs}} \tag{15}$$

The empirical FFRT failure rate is then defined as,

$$P_{f,FFRT} = \frac{\text{\# of accepted and incorrectly fixed epochs}}{\text{total \# of epochs}} \tag{16}$$

This empirical failure rate should theoretically be at most or equal to the user-defined failure-rate. The empirical probability of successful fixing can be computed as (12),

$$P_{sf,FFRT} = \frac{P_{s,FFRT}}{P_{s,FFRT} + P_{f,FFRT}} \tag{17}$$

Table 4: Empirical ILS success rate and FFRT statistics with user-defined $P_f = 0.1\%$. Number of epochs 14400.

System/ frequency	NO VALIDATION, empirical Success rate P_{s_E} [%]	Failure rate P_{f_E} [%]	WITH VALIDATION, FFRT empirical Successful fix $P_{sf,FFRT}$ [%]	Success rate $P_{s,FFRT}$ [%]	Failure rate $P_{f,FFRT}$ [%]
BeiDou					
B1	83.6	16.4	99.8	34.9	0.1
B2	91.9	8.1	99.6	57.9	0.2
GPS					
L1	90.2	9.8	100.0	45.1	0.0
L2	97.3	2.7	100.0	79.5	0.0
COMBINED					
B1+L1	100.0	0.0	100.0	100.0	0.0
B2+L2	100.0	0.0	100.0	100.0	0.0
BeiDou					
B1,B2	100.0	0.0	100.0	100.0	0.0
B1,B2,B3	100.0	0.0	100.0	100.0	0.0
GPS					
L1,L2	100.0	0.0	100.0	100.0	0.0
COMBINED					
B1,B2+L1,L2	100.0	0.0	100.0	100.0	0.0
B1,B2,B3+ +L1,L2	100.0	0.0	100.0	100.0	0.0

Table 5: Empirical FCRT statistics with $c = \frac{1}{2}$ and $c = \frac{1}{3}$. Number of epochs 14400.

System/ frequency	FCRT $c = \frac{1}{2}$ empirical Successful fix $P_{sf,c=\frac{1}{2}}$ [%]	Success rate $P_{s,c=\frac{1}{2}}$ [%]	Failure rate $P_{f,c=\frac{1}{2}}$ [%]	FCRT $c = \frac{1}{3}$ empirical Successful fix $P_{sf,c=\frac{1}{3}}$ [%]	Success rate $P_{s,c=\frac{1}{3}}$ [%]	Failure rate $P_{f,c=\frac{1}{3}}$ [%]
BeiDou						
B1	96.7	55.4	1.9	98.5	35.8	0.5
B2	98.7	72.1	1.0	99.4	54.1	0.3
GPS						
L1	98.5	70.0	1.0	99.5	51.5	0.3
L2	99.6	89.1	0.3	99.9	78.4	0.1
COMBINED						
B1+L1	100.0	100.0	0.0	100.0	100.0	0.0
B2+L2	100.0	100.0	0.0	100.0	100.0	0.0
BeiDou						
B1,B2	100.0	100.0	0.0	100.0	100.0	0.0
B1,B2,B3	100.0	100.0	0.0	100.0	100.0	0.0
GPS						
L1,L2	100.0	100.0	0.0	100.0	100.0	0.0
COMBINED						
B1,B2+L1,L2	100.0	100.0	0.0	100.0	100.0	0.0
B1,B2,B3+ +L1,L2	100.0	100.0	0.0	100.0	100.0	0.0

The abovementioned success rates and FFRT statistics are presented in Table 4, with an a priori user-defined failure rate set to $P_f = 0.1\%$. In Table 5 we give the corresponding Fixed Critical-value Ratio Test (FCRT) with standard values in (11) of $c = \frac{1}{2}$ and $c = \frac{1}{3}$ respectively. We replace the subscript 'FFRT' for the success rates we now use '$c = \frac{1}{2}$' or '$c = \frac{1}{3}$', respectively.

As expected, the single-frequency single-systems with the largest code noise (Table 3) and smallest wavelengths (Table 1) have the highest fractions of empirical failure rates P_{f_E}. For BeiDou B1 up to 16.4% of the epochs were wrongly fixed, which is more than GPS L1 (9.8% failure rate) due to the smaller number of available BeiDou satellites (Figure 2). Fortunately however the combined system shows very significant improvement in terms of empirical success rates P_{s_E} and failure rates P_{f_E}, where all epochs were successfully fixed in all cases. When using multiple-frequencies for the single-systems and the combined system we can also successfully fix all the epochs. This was all without integer validation.

With integer validation, the FFRT provides the highest number of successful fixes $P_{sf,FFRT}$ equal or close to

100% in all cases. Compare this number to the FCRT with values reaching below 97% for BeiDou B1 ($c = \frac{1}{2}$). The FFRT gives also a failure rate equal or smaller than the user-defined value of 0.1% in all cases (except for BeiDou B2 with 0.2%), and gives smaller failure rates compared to FCRT. This implies that the FFRT indeed gives the best protection against wrongly fixed ambiguities.

GPS AND BEIDOU POSITIONING RESULTS

In this section we evaluate the RPP and RTK positioning performance for a combined BeiDou+GPS system as well as for the single-systems separately. We only compared triple-frequency BeiDou results with that of dual-frequency GPS, since the third GPS frequency at this time (2012) only was available from PRNs 1, 24 and 25.

Relative point positioning (RPP) results

As the ambiguous phase measurements do not contribute to positioning in case of a single epoch of data, the code-only RPP results are the float solutions of the single-epoch RTK. In Table 6, the RPP results for B1 BeiDou, L1 GPS and B1+L1 combined are given (corresponding to the single-epoch float solutions of Figures 3-4). The results are given in local North, East and Up positioning errors as derived from comparing the estimated positions to precise benchmark coordinates.

All coordinate components improve significantly for the combined system with 27%, 34% and 28% in North, East and Up standard deviations respectively as compared to GPS. Corresponding values are 51%, 30% and 48% as compared to BeiDou. The standard deviation for North component and BeiDou is larger than for East, due to the satellite geometry and the large fraction of GEO satellites around the equator, see Figure 1.

RTK positioning results

All correctly fixed positioning results are corresponding to the empirical success rates in Table 4. Single-frequency RTK positioning results for B1 BeiDou, L1 GPS and the combined B1+L1 system, are depicted in Figure 3 with epoch-by-epoch (instantaneous) ambiguity resolution. The gray color corresponds to the single-epoch float solutions, green color to the corresponding instantaneous ambiguity resolved correctly fixed solutions, and finally red color to wrongly fixed solutions. Note the about two orders of magnitude improvement when going from wrongly fixed and float to correctly fixed solutions.

Figure 4 gives the corresponding positioning error histograms with (Top) float solution and (Bottom) correctly fixed solution. The empirical and formal theoretical normal distributions are based on the empirical mean and empirical and formal standard deviations of the position errors respectively, derived from the empirical and formal VCV-matrix of the positions. The empirical VCV-matrix is given by the positioning errors as derived from comparing the estimated positions to the precise benchmark coordinates, whereas the formal VCV-matrix is given as the mean of all single-epoch least-squares VCV-matrices of the position parameters of the entire observation span. The good match between the two distributions indicates realistic assumptions for the stochastic model in Table 3.

BeiDou+GPS single-frequency RTK empirical positioning statistics are given in Table 7. The corresponding dual-/triple-frequency results are given in Table 8. In Table 7 we see that the single-epoch based float solutions (which depend on the code noise) positioning standard deviations are overall larger for L1 and B1, due to the corresponding larger code noise as compared to B2 and L2 (see Table 3). For L1 and B1 we see for the combined system a 25% improvement in North, 30% in East, and 23% in Up empirical standard deviations as compared to GPS-only fixed solutions. The corresponding improvements as compared to BeiDou are 54%, 30% and 59% in North, East and Up respectively. In Table 8 we see an improvement for the standard deviations of dual-frequency combined system compared to that of GPS fixed solutions of approximately 20%, 26% and 20% in North, East and Up respectively, whereas for BeiDou the corresponding improvements are 58%, 39% and 61% respectively.

BeiDou satellite geometry and multipath effects

We can see in Figures 3-4 that the North component is worse than the East for BeiDou due to the satellite geometry. This is further verified by Figure 5, where we plot the number of satellites (Top), correctly fixed BeiDou B1 positioning errors (Middle), and North, East and Up fixed solution formal standard deviations (Bottom). We restrict all days to track the same satellites to have comparable results between the days.

Note the corresponding variations between the formal standard deviations and the correctly fixed solution positioning errors. This formal standard deviation variation in combination with effects such as multipath is believed to cause the periodic behavior of the positioning errors that are visible in Figure 3 and 5, particularly for the Up-component. The behavior is also repeated between days in Figure 5. However for the combined system these effects are mitigated significantly.

We therefore give in Figure 6 the positioning errors based on day-differences, and formulate error histograms as well. By performing these day differences with a time separation of 23 hours and 56 minutes, similar to the GPS constellation repeatability period [37], multipath effects can be significantly reduced. The IGSO satellites have

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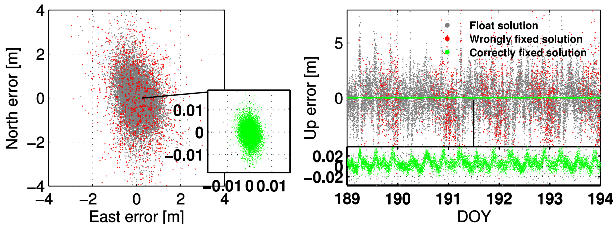
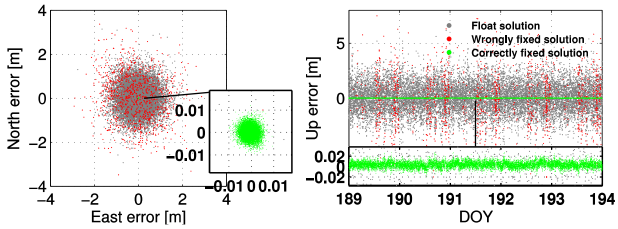
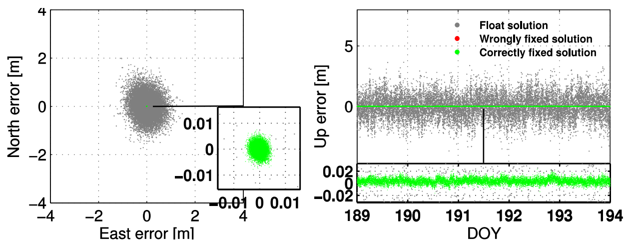


(a) **B1 BeiDou**.

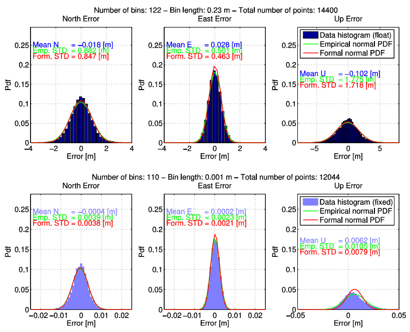


(b) **L1 GPS**.

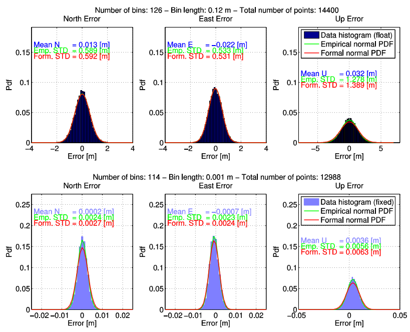


(c) **B1+L1 Combined**.

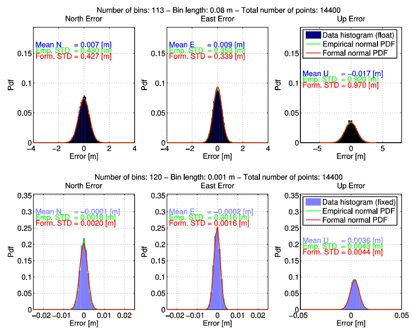
Figure 3: **B1+L1 BeiDou+GPS** single-epoch float (gray), correctly fixed (green) and wrongly fixed (red) positioning.



(a) **B1 BeiDou**.



(b) **L1 GPS**.



(c) **B1+L1 Combined**.

Figure 4: **B1+L1 BeiDou+GPS** single-epoch float (top) and correctly fixed (bottom) positioning histograms.

Table 6: B1+L1 RPP for BeiDou/GPS and combined. Positioning results are for July 7-11, 2012. Improvement of the combined system is compared to GPS, and in brackets compared to BeiDou.

System/frequency	Mean error N [m]	STD N [m]	Mean error E [m]	STD E [m]	Mean error U [m]	STD U [m]	Improvement STD [%]		
							N	**E**	**U**
BeiDou B1	-0.018	0.882	0.028	0.501	-0.102	1.775	-	-	-
GPS L1	0.013	0.589	-0.022	0.533	0.032	1.278	-	-	-
COMBINED B1+L1	0.007	0.430	0.009	0.353	-0.017	0.920	27 (51)	34 (30)	28 (48)

Table 7: BeiDou/GPS and combined **single-frequency** RTK positioning results for July 7-11, 2012. Improvement of the combined system is compared to GPS, and in brackets compared to BeiDou.

System/frequency	Mean error N [m]	STD N [m]	Mean error E [m]	STD E [m]	Mean error U [m]	STD U [m]	Improvement STD [%] N	E	U
BeiDou B1 float	-0.018	0.882	0.028	0.501	-0.102	1.775	-	-	-
BeiDou B2 float	0.008	0.813	0.101	0.523	0.142	1.597	-	-	-
GPS L1 float	0.013	0.589	-0.022	0.533	0.032	1.278	-	-	-
GPS L2 float	-0.007	0.433	0.041	0.410	-0.016	0.978	-	-	-
COMBINED B1+L1 float	0.007	0.430	0.009	0.353	-0.017	0.920	27 (51)	34 (30)	28 (48)
COMBINED B2+L2 float	-0.009	0.349	0.064	0.319	0.040	0.773	19 (57)	22 (39)	21 (52)
	Mean error N [mm]	**STD N [mm]**	**Mean error E [mm]**	**STD E [mm]**	**Mean error U [mm]**	**STD U [mm]**	**Improvement STD [%] N**	**E**	**U**
BeiDou B1 fixed	-0.4	3.9	0.2	2.3	6.2	10.5	-	-	-
BeiDou B2 fixed	-0.5	4.1	-0.5	2.5	5.5	11.0	-	-	-
GPS L1 fixed	0.2	2.4	-0.7	2.3	3.6	5.6	-	-	-
GPS L2 fixed	0.8	2.5	-0.9	2.4	4.3	6.0	-	-	-
COMBINED B1+L1 fixed	-0.1	1.8	-0.2	1.6	3.6	4.3	25 (54)	30 (30)	23 (59)
COMBINED B2+L2 fixed	0.4	2.0	-0.7	1.6	3.9	4.6	20 (51)	33 (36)	23 (58)

Table 8: BeiDou/GPS and combined **multiple-frequency** RTK positioning results for July 7-11, 2012. Improvement of the combined system is compared to GPS, and in brackets compared to BeiDou.

System/frequency	Mean error N [m]	STD N [m]	Mean error E [m]	STD E [m]	Mean error U [m]	STD U [m]	Improvement STD [%] N	E	U
BeiDou B1,B2 float	-0.005	0.586	0.066	0.354	0.018	1.175	-	-	-
BeiDou B1,B2,B3 float	0.078	0.517	0.002	0.305	0.016	0.998	-	-	-
GPS L1,L2 float	0.002	0.377	0.010	0.349	0.007	0.832	-	-	-
COMBINED B1,B2+L1,L2 float	-0.001	0.285	0.038	0.242	0.019	0.623	24 (51)	31 (32)	25 (47)
COMBINED B1,B2,B3+L1,L2 float	0.029	0.271	0.009	0.225	-0.019	0.583	28 (48)	36 (26)	30 (42)
	Mean error N [mm]	**STD N [mm]**	**Mean error E [mm]**	**STD E [mm]**	**Mean error U [mm]**	**STD U [mm]**	**Improvement STD [%] N**	**E**	**U**
BeiDou B1,B2 fixed	-0.4	3.8	-0.1	2.3	5.8	10.2	-	-	-
BeiDou B1,B2,B3 fixed	-0.3	3.8	-1.1	2.2	4.8	10.5	-	-	-
GPS L1,L2 fixed	0.5	2.0	-0.8	1.9	3.9	5.0	-	-	-
COMBINED B1,B2+L1,L2 fixed	0.1	1.6	-0.5	1.4	3.7	4.0	20 (58)	26 (39)	20 (61)
COMBINED B1,B2,B3+L1,L2 fixed	0.1	1.7	-1.0	1.5	3.3	4.2	15 (55)	21 (32)	16 (60)

same repeatability as GPS [38], and the GEO satellites are (almost) stationary. It is however still an approximation as different satellites can have different repeatability period shifts other than 4 minutes [37]. Any gaps visible in the time-series are due to wrongly fixed ambiguities for one/both of the days.

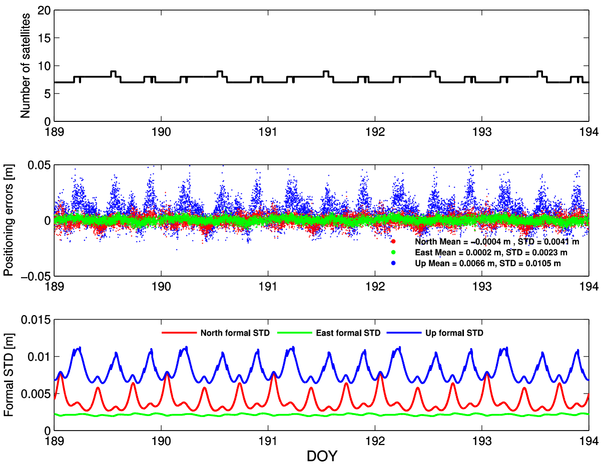


Figure 5: **B1 BeiDou** (Top) Number of satellites, (Middle) correctly fixed solution positioning errors, and (Bottom) North, East and Up fixed solution formal standard deviations for single-baseline RTK.

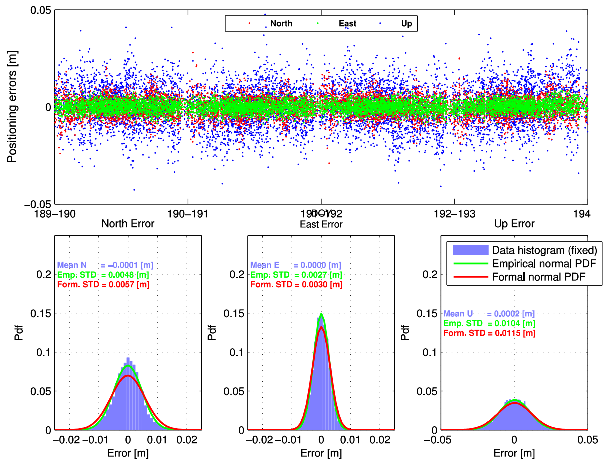


Figure 6: **B1 BeiDou day-differences** (Top) correctly fixed solution positioning errors, and (Bottom) corresponding fixed solution histograms for single-baseline RTK.

The periodic behavior has now in principle been eliminated, the mean errors are close to zero, and the corresponding histograms of the positioning errors fit well with the empirical theoretical normal distribution. Compare these histograms with Figure 4 and without day-differences (with multipath effects), where the histogram of the Up-component and BeiDou does not fit equally well with the empirical theoretical normal distribution.

CONCLUSIONS

BeiDou is the third satellite system (in addition to GPS and GLONASS) that offers continuous navigation in the Asia-Pacific region. In this contribution we have analyzed relative code-only positioning (RPP) and instantaneous (single-epoch) RTK positioning performance of a combined BeiDou+GPS system, and for the systems separately. We summarize our main findings and conclusions as follows.

Relative Point Positioning

For single-system, single-frequency RPP one can expect standard deviations below 1 m in the horizontal and $1-2$ m in the vertical component, whereas for the combined system corresponding values are below 0.5 m in the horizontal and below 1 m in the vertical component respectively. For BeiDou, the North component is less precisely estimable as compared to the East component. This is due to the large fraction of satellites located to the North and along the equator (four GEO satellites), with only a few satellites to the South of our Perth stations (Figure 1). Would all satellites be located on a cone, then the position in the direction of the cone's symmetry axis would be undetermined, see e.g. [39].

Integer ambiguity success rates

The single-frequency, single system empirical success rates (Table 4) were in the range $84-97\%$. For the GPS+BeiDou combined system, however, these success rates dramatically improved to 100%, primarily due to the increase of the number of satellites. This was all without integer validation. With validation, the FFRT provided us with almost 100% successful fixes for all these cases, which implies that it indeed makes a correct decision most of the time. More specifically, we can compare the FFRT failure rate of at most 0.2% to that of the traditional Fixed Critical-value Ratio Test (FCRT) (Table 5) of up to 2%, and thus conclude that FFRT indeed yields better protection against wrongly fixed ambiguities.

RTK positioning

The instantaneous RTK positioning results showed that the fixed solution positioning standard deviations improvements for single-frequency and a combined system were on average, for all coordinate components, approximately 26% as compared to GPS. Corresponding value was 48% as compared to BeiDou (Table 7). The dual-frequency (Table 8) corresponding improvements were 22% and 53% for GPS and BeiDou respectively. All empirical standard deviations of correctly fixed solutions are at the mm-cm level, whereas the precision of incorrectly fixed solutions can become even worse than the corresponding float solution at the meter-level (Figure 3). For single-frequency RTK only the combined system provided for correctly fixed solutions continuously over the days (Tables 4-5).

More BeiDou MEO satellites are upcoming and will further improve the current satellite geometry. We can then expect more positioning and success rates improvements for BeiDou and the combined GPS+BeiDou system.

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